

See-Saw Modification of Gravity

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Abstract

We discuss a model in which the fundamental scale of gravity is restricted to 10^{-3} eV. An observable modification of gravity occurs simultaneously at the Hubble distance and at around 0.1 mm. These predictions can be tested both by the table-top experiments and by cosmological measurements. The model is formulated as a brane-world theory embedded in a space with two or more infinite-volume extra dimensions. Gravity on the brane reproduces the four-dimensional laws at observable distances but turns to the high-dimensional behavior at larger scales. To determine the crossover distance we smooth out the singularities in the Green's functions by taking into account softening of the graviton propagator at energies above the fundamental scale. This is due to the high-dimensional operators which are suppressed by the fundamental scale. We find that irrespective of the precise nature of microscopic gravity the ultraviolet and infrared scales of gravity-modification are rigidly correlated. This fixes the fundamental scale of gravity at 10^{-3} eV. The result persists for nonzero thickness branes.

1 Introduction

The law of gravitational interactions is tested experimentally at distances above 0.1 millimeter [1]. Hence, gravity could change its behavior at a scale which is as low as:

$$M_*^{\text{exp}} \sim (0.1 \text{ mm})^{-1} \sim 10^{-3} \text{ eV} . \quad (1)$$

A theory of gravitation which predicts a deviation from the conventional law above M_* , such that gravity is still negligible above this scale compared to the gauge interactions, cannot be ruled out at present.

Another important piece of data which is relevant for gravity comes from recent astrophysical measurements. The data indicate that the expansion of the universe might be accelerating [2]. The existence of the cosmic acceleration raises yet another interesting question on possible infrared (IR) modification of gravity. Indeed, an alternative view might be that the observed acceleration of the universe is not because of the vacuum energy, but due to the modification of the laws of gravity at the Hubble distance. Therefore, there might be another scale in the theory of gravitation

$$H_0^{-1} \sim 10^{29} \text{ mm} , \quad (2)$$

where H_0 denotes the present-day value of the Hubble parameter.

The aim of the present work is to discuss a model which predicts the modification of gravity at distances of order 10^{29} mm and as a consequence of this, the modification at 0.1 mm ensues. As we will see, these two scales are intimately related. As a result, the model restricts the value of the fundamental scale of gravity to 10^{-3} eV.

A brane-world model, in which extra dimensions have an infinite-volume and the 4D laws of gravity are reproduced on a brane due to the induced Einstein-Hilbert term, predicts the modification of gravity at very large cosmological distances [3]. The 4D Einstein-Hilbert term “shields” a brane observer from strong bulk gravity [4]. Consequently, the observer detects the conventional 4D gravity all the way up to large cosmological distances. At those scales the effects of infinite-volume extra dimensions take over and the behavior of gravity changes. In a 5D model one finds the following crossover scale [3]:

$$r_c^{(D=5)} \sim \frac{M_{\text{Pl}}^2}{M_*^3} , \quad (3)$$

where M_{Pl} is the 4D Planck mass and M_* denotes the fundamental gravitational scale of a 5D bulk theory. For large distances, $r \gg r_c^{(D=5)}$, gravity is five-dimensional, while it is four-dimensional when $r \ll r_c^{(D=5)}$. The laws of the 4D gravity are valid all the way down to the distances of order $1/M_*$. However, below this length scale the effective theory of gravity breaks down. As a result, there is a lower bound on the scale M_* which comes from the accelerator, astroparticle, and cosmological data, that is $M_* \gtrsim 10^{-3}$ eV [4].

Therefore, one finds the laws of 4D gravity only in the interval:

$$\frac{1}{M_*} \lesssim r \lesssim \frac{M_{\text{Pl}}^2}{M_*^3}. \quad (4)$$

This scenario leads to novel astrophysical and cosmological consequences and predictions which were found in Refs. [5–14]. In fact, it allows for the accelerated universe with no cosmological constant which is consistent with the data [7]. Some of these results will be discussed below. A similar phenomenon for photons was studied in Ref. [15] (see also Ref. [16]). In these models the behavior of gravity and gauge fields are alike.

Note that in order for the large distance modification of gravity to have relevance to present day cosmology and astrophysics, the crossover scale should be at or below the observable size of the universe, $r_c^{(D=5)} \lesssim 10^{29}$ mm. This imposes the following restriction $M_* \gtrsim 10$ MeV. Therefore, in this scenario, gravity will simultaneously be modified at the horizon size and at $(10 \text{ MeV})^{-1} \sim 20$ Fermi.

In the present paper we will argue that the higher dimensional ($D \geq 6$) generalization [5] gives rise to a different dependence of the crossover scale on M_{Pl} and M_* . In particular, for the modification of gravity at around 10^{29} mm we get $M_* \sim 10^{-3}$ eV, and *vice versa*. The fundamental scale of gravity is bound to be 10^{-3} eV. Thus, this model makes very restrictive predictions on the modification of gravity which are testable in table-top experiments and simultaneously can be testable in astrophysical and the cosmological measurements.

There are additional reasons why this model can be interesting to study¹. The scale 10^{-3} eV is very close to the scale of the vacuum energy density [2]. Although, this similarity might be a coincidence, nevertheless, it is rather intriguing to argue, following Ref. [19], that the scale of the vacuum energy density might be related to the ultraviolet cutoff of the gravitational theory. Independently of the above argument, the theories with infinite-volume extra dimensions might shed new light to the cosmological constant problem [20,21], [5] (for discussions of these models in string theory see Refs. [22–24]).

The generalization to the $D = (4 + N)$ -dimensional models with $N \geq 2$ is not completely trivial, however. Certain additional singularities emerge in this case [5] and they should be dealt with some care. The presence of these singularities is related to the fact that the Green's function of the d'Alembertian in the space transverse to a 3-brane blows up at the origin for $N \geq 2$, while it is regular for $N = 1$. One of the purposes of the present paper is to deal carefully with these singularities.

In what follows we will cast the singularities into the following two classes.

- Type I: These are the singularities which are related to the fact that the brane is approximated by a delta-function type source. They are present as in the $N = 1$, as well as in $N \geq 2$ cases.

¹For models with warped geometries and modification of gravity at large scales see Refs. [17,18].

- Type II: These are the singularities which are present for the $N \geq 2$ case; they are related to the fact that the transverse Green’s function of the d’Alambertian in $N \geq 2$ dimensional space blows up at the origin.

Although these singularities have a common origin, nevertheless, it is convenient to discuss them separately in order to compare the $N = 1$ and $N \geq 2$ models.

Let us start with the Type I singularities, i.e., those associated with a vanishing brane width. Suppose the brane had a finite thickness (we call it Δ)². What would be the effect of this width on a low energy observer in the 4D worldvolume theory? One effect is the presence of a massive state (or states) which correspond to the fluctuation of the brane transverse width (the “breathing mode”). The mass of this state is of the order the inverse brane width. This state, being localized and gravitationally interacting mode, will certainly renormalize the graviton kinetic term on the 4D worldvolume, so that the generic value of the coefficient in front of the 4D graviton induced term is of the order Δ^{-1} . Other than that, the presence of such a heavy state is irrelevant for low-energy 4D physics³. Therefore, from the point of view of a low-energy observer this state can be “integrated out”. This is equivalent of “integrating out” the brane width. Hence, in the low-energy theory the brane will look like a delta-function source. Since we expect that the physics is smooth and continuous, the calculations at low energies, i.e., at $E \ll \Delta^{-1}$, in a theory with the delta-function brane should produce the same result (up to the $\mathcal{O}(E\Delta)$ corrections) as calculations in a theory where the ultraviolet resolution of the brane is present due to a nonzero brane width. This is indeed what we will find below.

Let us now turn to Type II singularities. They will be taken care of separately from the singularities associated with the delta function brane. In fact, as a first step we could keep the brane as a delta-function source and try to remove the type II singularities. For this one could introduce for instance the “ ϵ shift regularization” as it was done in [5], or one could use the rigid cutoff in the bulk as in Ref. [25]. However, a more justified way to deal with this problem is to introduce HD operators as was proposed in Refs. [22] and [4]. From the physical standpoint, the HD’s are remnants of UV physics which is generically unknown. Thus, it is natural that they provide regularization of certain UV singularities in the low-energy theory. This is the way which we will follow in the present paper.

The paper is organized as follow. In section 2 we study the induced terms on a thick brane. We will show that these terms in general are nonlocal and have no factorizable form in terms of the worldvolume and transverse coordinates. These nonlocal expressions, however, can be expanded in an infinite series of local terms. The expansion parameter is the width of the brane. The leading term coincides with the 4D Einstein-Hilbert term. In section 3 we setup a scalar field theory model which

²Below in sections 3-6 we will be dealing with a brane the transverse width of which is much less than the fundamental distance scale in the bulk. Such an unusual brane can exist even in 4D theory, see discussions below and in section 3 of Ref. [4].

³It also renormalizes the brane tension.

mimics the properties of a theory with the induced kinetic term on a thick brane. At this point we fine-tune the brane tension and the 4D cosmological constant to make the calculation analytically treatable. In section 4 we use this model to calculate the two-point Green's function. First we consider the delta-function brane but introduce the HD operator which smoothes out the Type II singularities in the expression for the Green's function. We show that the behavior of the gravity is qualitatively similar to that of the 5D theory: at short distances the theory behaves as four-dimensional and at large distances it approaches the $(4 + N)$ -dimensional regime. The expression for the crossover distance in this case is different from the one found for the 5D theory. In section 5 we consider the case when the brane width is kept fixed and calculate the corresponding Green's functions. We show that the qualitative behavior described above holds unchanged. The limiting transition to the delta-function brane is smooth. There are no ultra-local interactions [26] in the bulk and no new physical scales [22] arise in these calculations. In section 6 we discuss the phenomenological constraints on the crossover scale beyond which gravity can change its regime. In section 7 we consider a brane with a nonzero tension and give an estimate for the crossover scale in this case. Conclusions are presented in section 8.

2 Induced Terms on Smooth Branes

In this section we study how the induced terms arise on a smooth solitonic brane. We will argue that in the approximation when the brane width is kept finite the induced terms are nonlocal and not factorizable with respect to worldvolume and transverse dimensions. However, in the low-energy approximation they can be expanded in a power series of local terms with the leading term coinciding with the 4D Einstein-Hilbert action.

Our consideration in this section is rather general and applies to any brane-world model (with a non-conformal-invariant worldvolume theory). Let us start with a $(4 + N)$ -dimensional action which contains a graviton G_{AB} and a scalar field Π which makes the brane, and other fields Ψ which do not participate in the formation of the classical background⁴:

$$S = \int d^4x d^N y \sqrt{G} \mathcal{L}(G, \Pi, \Psi) . \quad (5)$$

Here \mathcal{L} denotes the total Lagrangian density which is a function of the fields and their derivatives which are suppressed in (5). The bulk graviton indices are decomposed as follows:

$$G_{AB}(x_\mu, y_i), \quad A, B = 0, 1, \dots, 3 + N; \quad \mu, \nu = 0, 1, 2, 3; \quad i, j = 4, 5, \dots, 3 + N . \quad (6)$$

⁴All the derivations in this section can be generalized straightforwardly for more complicated classical backgrounds.

We study the Lagrangian which give rises to a classical solution in a form of a three-brane. This solution depends on the coordinates y_i but does not depend on x_μ . Many examples of this are known in various dimensions. Let us split the graviton and scalar fields in their classical parts and fluctuations:

$$G_{AB} = G_{AB}^{\text{cl}}(y) + H_{AB}(x, y), \quad G_{\mu\nu}^{\text{cl}}(y) \equiv A^2(y) \eta_{\mu\nu}, \quad G_{\mu j}^{\text{cl}} = 0, \quad (7)$$

$$\Pi(x_\mu, y_i) = \Pi^{\text{cl}}(y) + \sigma(x_\mu) f(y). \quad (8)$$

Where in (7) we use the parametrisation of the $\{\mu\nu\}$ component of the background metric in terms of the function $A^2(y)$ which is typical for the branes which preserve the 4D Poincare invariance on the worldvolume.

The scalar field Π has modes which are localized on a brane. The most obvious ones are the Goldstone bosons associated with spontaneously broken translation invariance in the y_i directions. These particles are massless and are derivatively coupled to matter fields on a brane. For these reasons they are not relevant for our discussions below. In addition, generically there are massive localized modes on a brane. For simplicity we consider below only a single mode. As we mentioned before, the latter corresponds to the fluctuations of the transverse width of the brane. Thus, its mass is proportional to the inverse brane width. Let us discuss this mode in detail. Since it is localized on a brane there should exist an effective four-dimensional Lagrangian for it. To derive this Lagrangian let us start with the action for Π :

$$\int d^4x d^N y \sqrt{G} \left\{ \frac{1}{2} G^{AB} \partial_A \Pi \partial_B \Pi - V(\Pi) \right\}. \quad (9)$$

Here V is a potential which is responsible for the existence of the brane. Let us now substitute (7) and (8) into (9) and keep truck of quadratic fluctuations (interactions will be discussed later). In this approximation

$$\int d^4x d^N y \sqrt{G^{\text{cl}}(y)} \left\{ \frac{f^2(y)}{A^2(y)} \frac{\partial_\alpha \sigma(x) \partial^\alpha \sigma(x)}{2} - \frac{\sigma^2}{2} [f^2 V''(\Pi^{\text{cl}}) - \partial_i f \partial^i f] \right\}. \quad (10)$$

(Here prime denotes differentiation w.r.t. Π . The 4D indices here and below are contracted by the flat space metric $\eta_{\mu\nu}$ and those of the extra coordinates by G_{ij}^{cl} .) The quadratic term in σ is a mass term. The value of the mass depends on the exact expression for f . For instance, by definition of a localized Goldstone particle, its profile f is such that the mass term in (10) vanishes. On the other hand, the profile f for a breathing mode is different. In fact, the function f for a breathing mode satisfies the equation ⁵:

$$f^2 V''(\Pi^{\text{cl}}) + \frac{f \partial_i \left(\sqrt{G^{\text{cl}}(y)} \partial^i f \right)}{\sqrt{G^{\text{cl}}(y)}} = \frac{M^2 f^2}{A^2}. \quad (11)$$

⁵Note that functions f for different modes are orthogonal to each other, that is why we can treat them separately.

Here the parameter M is proportional to the inverse brane width:

$$M \sim \frac{1}{\Delta} . \quad (12)$$

Using (11) we obtain the following expression for the quadratic part of the low energy 4D Lagrangian of a breathing mode:

$$\left[\int \sqrt{G^{\text{cl}}(y)} \frac{f^2(y)}{A^2(y)} d^N y \right] \times \int d^4 x \left\{ \frac{1}{2} \partial_\alpha \sigma(x) \partial^\alpha \sigma(x) - \frac{1}{2} \sigma^2 M^2 \right\} . \quad (13)$$

We see that the y dependence can be integrated out. This is because for large y the localization function for local defects is exponentially decreasing, ($\sim \exp(-|y|/\Delta)$). This has to make the y integral in (13) to converge for a breathing mode even though the integrals

$$\int \sqrt{G^{\text{cl}}(y)} d^N y \quad \text{and} \quad \int \sqrt{G^{\text{cl}}(y)}/A^2(y) d^N y$$

might not be convergent (the latter case corresponds to the brane models where there is no localized graviton zero mode).

Let us now turn to the gravitational part of the action. We will consider a general case when the graviton is not necessarily localized. If so, the bulk gravity action cannot be integrated w.r.t. y to obtain 4D kinetic term for 4D gravitons. Nevertheless, the 4D laws of gravity can be obtained on a brane due to the induced terms [3, 5] to the discussion of which we turn now.

Let us look at the interaction of bulk gravity with the localized field σ . We concentrate first on the $\{\mu\nu\}$ components of the interactions since these are the ones that determine the 4D induced terms relevant to us. Using (7),(8) and (9) we derive the interaction Lagrangian for the $\{\mu\nu\}$ part:

$$\int d^4 x d^N y \sqrt{G^{\text{cl}}(y)} \frac{f^2(y)}{A^4(y)} \eta^{\alpha\mu} \eta^{\beta\nu} H_{\alpha\beta}(x, y) \times \left\{ \partial_\mu \sigma(x) \partial_\nu \sigma(x) - \eta_{\mu\nu} \left[\frac{1}{2} \partial_\lambda \sigma(x) \partial^\lambda \sigma(x) - \frac{1}{2} \sigma^2 M^2 \right] \right\} . \quad (14)$$

The quantity in second line in (14) is nothing but the energy momentum tensor for σ with the mass M . From this we see that the interaction of a bulk graviton with the localized mode can be rewritten in a purely 4D form. Indeed, rescaling the field $\sigma \rightarrow \sigma / (\int d^N y \sqrt{G^{\text{cl}}(y)} f^2(y)/A^2(y))^{1/2}$ we get the canonically normalized 4D kinetic term for the field σ in (13) and its interaction with the bulk gravity, according to Eq. (14), takes the following form:

$$S_{\text{int}} = \int d^4 x h^{\mu\nu}(x) T_{\mu\nu}^\sigma(x) , \quad (15)$$

where the field h is defined as follows:

$$h_{\mu\nu}(x) = \frac{\int d^N y [\sqrt{G^{\text{cl}}(y)} f^2(y) H_{\mu\nu}(x, y)/A^4(y)]}{\int d^N y [\sqrt{G^{\text{cl}}(y)} f^2(y)/A^2(y)]} . \quad (16)$$

Thus, the interaction Lagrangian of higher dimensional gravity with a localized mode can be given a purely 4D form (15) in spite of the fact that the bulk graviton might not be localized on a brane (i.e., the bulk graviton kinetic term might not be integrable w.r.t. y). In what follows we will study loop effects due to this interaction.

Before we turn to these discussions we would like to digress for a moment and make two comments. So far we discussed only a single localized field. Suppose now there are n localized fields σ_k , $k = 1, 2, \dots, n$, with the localization functions f_k which in general can be different. Then, the interaction vertex of each of this state with the bulk gravity can be presented in the form of Eq. (15). On the other hand, since f 's are different and the expressions for f 's determine $h_{\mu\nu}(x)$ in (16), then each of these localized states will interact with different “effective 4D gravitons” (16). However, these effects are suppressed by the brane width and are negligible as we will see below. For this let us perform the expansion:

$$H_{\mu\nu}(x, y) = H_{\mu\nu}(x, 0) + y^i \partial_i H_{\mu\nu}(x, 0) + \frac{1}{2} y^i y^j \partial_i \partial_j H_{\mu\nu}(x, 0) + \dots \quad (17)$$

We substitute this series into (16) and then in (15) for each localized field. As a result, for each field we get infinite number of terms first two of which are:

$$H^{\mu\nu}(x, 0) T_{\mu\nu}^{\sigma_k}(x) \int \sqrt{G^{\text{cl}}(y)} \frac{f_k^2(y)}{A^2(y)} d^N y, \quad (18)$$

$$\partial_i H^{\mu\nu}(x, 0) T_{\mu\nu}^{\sigma_k}(x) \int y^i \sqrt{G^{\text{cl}}(y)} \frac{f_k^2(y)}{A^2(y)} d^N y, \quad (19)$$

and so on, with increasing powers of derivatives. The first term (18) gives the universal interaction for all the different fields. This can be seen by performing the rescaling of the fields to bring their kinetic terms to the canonical form in (13). Then, the couplings of all the localized fields to $H_{\mu\nu}(x, 0)$ are identical. In fact, the result of this universal term is what we obtain in the delta-function limit for the brane, i.e., when $f^2 \rightarrow \delta^{(N)}(y)$. This is the dominant contribution. The higher terms, on the other hand, are not like this. Each higher term (including the one in (19)) can be thought of as a new field $\partial_i \partial_j \dots \partial_k H_{\mu\nu}(x, 0)$ from the 4D point of view. However, as we can see from (19), the couplings of these fields to the energy-momentum tensor contain extra powers of y under the integral. Since the effective region of integration is determined by the “width” of the function f , then each additional power of y will translate into an additional power of Δ . Therefore this couplings are suppressed by extra powers of the brane width Δ . For instance, the term in (19) can give rise to the contributions containing extrinsic curvature and its derivatives. However, these terms are suppressed compared to the leading ones by the powers of the brane width⁶.

⁶In Ref. [23] it was found that on a D-brane in bosonic string perturbation theory the induced extrinsic curvature terms are of the same order as the Einstein-Hilbert term and both are suppressed by powers of string coupling constant. However, the nonperturbative effects, e.g., those due to

Therefore, we conclude that in the low-energy theory the operators which would violate universality are suppressed by powers of Δ . The brane width has to be small enough, at the scale $1/M_{\text{Pl}}$ for instance, to leave this effects unobservable.

After this digression we return back to the case of one localized scalar field and try to study it as far as possible.

Having the interaction vertices derived let us look at the loop diagrams where two external graviton lines are attached to the loops in which the localized fields are running. The vertices in these diagrams are described by (15). One can think of this interaction as purely 4D interaction of $T_{\mu\nu}^\sigma(x)$ with the effective 4D graviton $h_{\mu\nu}(x)$. These loop diagrams with two external $h_{\mu\nu}$ lines give rise to the following term in the effective action on a brane:

$$\tilde{h}_{\mu\nu}(p) \hat{\mathcal{O}}^{\mu\nu\alpha\beta}(p, M) \tilde{h}_{\alpha\beta}(-p) , \quad (20)$$

where $\hat{\mathcal{O}}^{\mu\nu\alpha\beta}(p, M)$ is some function of the external momentum p and the particle mass M . In the one loop approximation this function can be expressed in terms of hypergeometric functions (see e.g, [27]). This leads to a nonlocal interactions in the effective Lagrangian in the coordinate space. The explicit form of the nonlocal coordinate-space Lagrangian is hard to present. However, at the momenta $p \ll M$ we can perform an expansion of the formfactor $\hat{\mathcal{O}}^{\mu\nu\alpha\beta}$ in powers of p^2/M^2 . The leading term takes the form:

$$M^2 \tilde{h}_{\mu\nu}(p) \left(\eta_{\mu\alpha}\eta_{\nu\beta}p^2 - \eta_{\mu\nu}\eta_{\alpha\beta}p^2 - \eta_{\mu\alpha}p_\nu p_\beta - \eta_{\nu\beta}p_\mu p_\alpha + \eta_{\alpha\beta}p_\mu p_\nu + \eta_{\mu\nu}p_\alpha p_\beta \right) \tilde{h}_{\alpha\beta}(-p) . \quad (21)$$

This is a quadratic approximation to a reparametrization invariant graviton kinetic term in a 4D low-energy effective theory of gravity (for treatment of general relativity as an effective field theory, see Ref. [28]).

Let us now turn to the $\{ij\}$ components. These will induce the brane-kinetic and mass terms for the components G_{ij} . The latter, look as scalars (“graviscalars”) from the point of view of a braneworld observer. However, the mass terms (potentials) of these states on the brane might not protected by any symmetries and become of the order the UV cutoff of the worldvolume theory. Such states cannot mediate forces which would compete with gravity at observable distances. In addition, even if they stay massless, they should decouple from the 4D matter at short (observable) distances in analogy with the phenomenon found in Ref. [12].

Finally, we turn to the $\{i\mu\}$ components. From a 4D perspective these are graviphotons. In the linearized approximation they are derivatively coupled to conserved currents. These terms cannot give rise to gravity competing forces at observable distances and are irrelevant for our considerations.

the inverse brane width studied above, are expected to give the dominant contribution to the worldvolume Einstein-Hilbert term. The terms with extrinsic curvature will in general be induced on a brane as well. It is interesting to study the effects of these terms.

3 A Scalar Field Model

To summarize the results of the previous section we write down the gravitational part of the action which includes the lowest dimensional derivative terms in the bulk and those induced on the brane:

$$S = M_*^{2+N} \int d^4x d^N y \sqrt{|G|} \mathcal{R}(G) + M_{\text{ind}}^2 \int d^4x \sqrt{|g|} R(g) + \text{other terms} . \quad (22)$$

Here, the induced 4D term should be understood as an expansion in perturbations defined as follows:

$$g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + h_{\mu\nu}(x) , \quad (23)$$

where $h_{\mu\nu}(x)$ is related to the bulk graviton G via (16)⁷. The first term in (22) is the bulk kinetic term for a graviton and the second term is a brane-induced kinetic term for the very same graviton.

In the previous section we found that in general the induced terms are nonlocal. The term which is given in (22) is just a first term in the expansion of that nonlocal expression in powers of the brane width Δ . However, as long as we discriminate between $h_{\mu\nu}(x)$ and $H_{\mu\nu}(x, 0)$, the induced term in (22) itself contains the effects of the same order, i.e., those which are proportional to Δ . This means that for any nonzero Δ the action (22) is not complete. The additional terms which vanish in the $\Delta \rightarrow 0$ limit are missing in (22). Nevertheless, the latter action can be used to check the limiting transition from the finite thickness brane to the delta function brane since in the limit $\Delta \rightarrow 0$ the missing terms do not contribute. Therefore, we will be able to check whether the action (22) gives rise to the results which in the limit $\Delta \rightarrow 0$ turn smoothly to the ones obtained with a delta function brane [3, 5, 15] answering some question raised in Refs. [22, 26].

Our goal is to study the interactions between sources which are located on the brane and which exchange gravitons described by the action (22). Furthermore, the value of the induced constant M_{ind}^2 is crucial. In order to reproduce the correct 4D gravity on a brane this has to equal to the 4D Planck mass, $M_{\text{ind}}^2 = M_{\text{Pl}}^2$ [3, 5]. The question is how does one get such a big scale on a brane when the bulks scale M_* is rather low. There are some known possibilities for this. First, there exist certain branes, even in 4D theories, the width of which is much smaller than the fundamental length scale in the bulk (see discussions on this in Sect. 3 of Ref. [4]). In this case, the induced constant will have the magnitude much bigger than the bulk scale M_* . Another possibility is that the brane worldvolume particle theory has very high UV scale (such as the GUT scale) and/or very high multiplicity of states which renormalize the worldvolume gravitational constant and make it of the order of M_{Pl} . In general, the induced scale is determined by a two-point correlation function of the energy-momentum tensor of the worldvolume particle theory which is localized on

⁷Note that the induced metric on the brane is actually $A^2(\Delta)\eta_{\mu\nu}$. The constant factor $A^2(\Delta)$ rescales M_{ind}^2 but is dropped for simplicity above. It will be taken into account when important in section 7.

the brane. This was discussed in detail in Refs. [4] (see also [5]). In either case, the 4D gravitational Planck scale is a derived quantity. Since the mechanism of this is not an emphasis of the present work, we will assume for simplicity that $\Delta^{-1} \sim M_{\text{Pl}}$ and that one of these mechanism leads to $M_{\text{ind}} = M_{\text{Pl}}$, which is a definition of M_{Pl} .

The next step is to calculate the two-point Green's functions for the gravitational action (22). This calculation can be straightforwardly performed for gravitons along the lines of [3], nevertheless, to give more clear derivation it is convenient to make the following simplification. In what follows we will suppress the tensorial structure since the latter does not change the issue of singularities. Although, there will be separate issues due to the tensorial structure of the Green's functions (such as the ones associated with perturbative discontinuity and nonperturbative continuity in $1/r_c$ advocated in Ref. [12]), however, since the physics of higher codimension cases is very similar to that of the 5D case, we expect that these issues can be addressed in a manner similar to what was found for the 5D case in Ref. [12] based on earlier work [29] (see also recent work [14]). Having this said, in what follows we substitute the graviton field by a scalar field Φ and study the Green functions for the latter. This allows to determine the force-law. Hence, the scalar Φ should be regarded as a counterpart of a graviton.

Until section 7 we will work with a brane which is placed at an orbifold fixed point and the tension of which is tuned to zero by means of the 4D cosmological constant. This can be applicable, for instance, to a non-BPS system of D-branes and Orientifold planes placed on top of each other at some fixed points. The generalization to the nonzero tension branes leads to some quantitative novelties as will be seen in section 7. Before that, we should put $\sqrt{G^{\text{cl}}} = 1$, $A(y) = 1$ and $\int f^2(y) d^N y = 1$ in the expressions obtained in the previous section.

The action for the scalar field which mimics the graviton with the induced kinetic term on a brane in a space with N extra dimensions is as follows:

$$S = \int d^4 x d^N y M_*^{2+N} \left[\partial_A \Phi(x, y) \partial^A \Phi(x, y) - \frac{\epsilon}{M_*^2} \Phi(x, y) \left(\partial_A \partial^A \right)^2 \Phi(x, y) \right] + M_{\text{Pl}}^2 \int d^4 x d^N y' d^N y'' f^2(y') \partial_\mu \Phi(x, y') f^2(y'') \partial^\mu \Phi(x, y'') , \quad (24)$$

Note that we have normalized the scalar field in such a way that it is dimensionless. The HD term, which is proportional to ϵ , is included for the regularization of the propagator (see below). The parameter ϵ is an arbitrary $\mathcal{O}(1)$ constant in general. Let us mention here that the results obtained in the next sections are not really bound to the form of the HD operator which we use. Any HD operator which smoothes out the UV effects will reproduce qualitatively the same results.

Below we will discuss two separate cases. First we approximate the brane by a delta function source and show that singularities are removed by the HD terms. In this case:

$$f^2(y) = \delta^{(N)}(y) . \quad (25)$$

As a next step, we consider a smooth finite thickness brane and study the tree-level propagator. For this we use the simplest ansatz:

$$f^2(y) = \begin{cases} \alpha^2, & \text{for } |y| < \Delta, \\ 0, & \text{for } |y| > \Delta, \end{cases} \quad (26)$$

where in the limit $\Delta \rightarrow 0, \alpha^2 \rightarrow \infty$ and $\alpha^2 \Delta^N = \text{const.}$

4 A Delta Function Brane

In this section we restrict ourselves to the consideration of a delta-function brane. The singularities will be removed by means of the HD operators. Thus, we set the expression (25) for the localization function and use the method of Ref. [5]. The equation for the Green function for a source located on the brane takes the form

$$\left(M_*^{2+N} \left[\partial_A \partial^A + \frac{\epsilon}{M_*^2} (\partial_A \partial^A)^2 \right] + M_{\text{Pl}}^2 \delta^{(N)}(y) \partial_\mu \partial^\mu \right) G(x, y; 0, 0) = \delta^{(4)}(x) \delta^{(N)}(y). \quad (27)$$

To find the solution of Eq. (27) let us turn to the Fourier-transformed quantities with respect to the worldvolume four-coordinates x_μ :

$$G(x, y; 0, 0) = \int \frac{d^4 p}{(2\pi)^4} e^{ipx} \tilde{G}(p, y).$$

Then, in the Euclidean worldvolume space Eq. (27) takes the form

$$\left(M_*^{2+N} \left[(p^2 - \partial_y^2) + \frac{\epsilon}{M_*^2} (p^2 - \partial_y^2)^2 \right] + M_{\text{Pl}}^2 \delta^{(N)}(y) p^2 \right) \tilde{G}(p, y) = \delta^{(N)}(y), \quad (28)$$

where $p = \sqrt{p_1^2 + p_2^2 + p_3^2 + p_4^2}$ denotes the magnitude of the Euclidean momentum.

We are looking for the solution in the following form

$$\tilde{G}(p, y) = D(p, y) B(p),$$

where $D(p, y)$ is defined as follows

$$\left(p^2 - \partial_y^2 + \frac{\epsilon}{M_*^2} (p^2 - \partial_y^2)^2 \right) D(p, y) = \delta^{(N)}(y). \quad (29)$$

$B(p)$ is some function which should be determined. Using the above expression one finds [5]:

$$\tilde{G}(p, y) = \frac{D(p, y)}{M_{\text{Pl}}^2 p^2 D(p, 0) + M_*^{2+N}}, \quad (30)$$

where $D(p, y)$ is given by

$$D(p, y) = \int \frac{d^N q}{(2\pi)^N} \frac{\exp(iqy)}{p^2 + q^2 + \epsilon (p^2 + q^2)^2 / M_*^2}.$$

Note that for $N \geq 2$ the ϵ/M_*^2 term smoothes out the UV divergences in the expression for $D(p, y)$. Indeed, without this term the expression for $D(p, y)$ blows up at $y = 0$. This divergence is not related to the presence of the delta function brane as it is clear from Eq. (29). Thus we keep the HD term (In the $N = 1$ case, the expression for $D(p, 0)$ is finite even without the HD term [3, 15, 5].)

As an example of a theory with higher than one codimensions, we consider the case of $N = 3$. The corresponding calculation can be done exactly. The results for $N \geq 2$ are similar ⁸.

For $N = 3$ the function $D(p, y)$ is given by

$$D(p, y) = \frac{1}{4\pi|y|} \{ \exp(-p|y|) - \exp(-|y|\tilde{p}) \} ; \quad \tilde{p} \equiv \sqrt{p^2 + \frac{M_*^2}{\epsilon}}. \quad (31)$$

At $y = 0$, we find

$$D(p, 0) = \frac{1}{4\pi} \left(\sqrt{p^2 + \frac{M_*^2}{\epsilon}} - p \right) \simeq \frac{1}{4\pi} \left(\frac{M_*^2}{\epsilon} \right)^{\frac{1}{2}} \left(1 - p \left(\frac{\epsilon}{M_*^2} \right)^{\frac{1}{2}} \right), \quad (32)$$

which is finite. Note that, an action with HD terms has a consistent interpretation only if it is regarded as an infinite series in high derivatives. Any truncation to a finite order can give rise to ghosts. Therefore, the HD term should be treated as giving a (p^2/M_*^2) correction in the expansion. Hence, the above expression for $D(p, y)$ makes sense only as an expansion in (p^2/M_*^2) in which only the first correction is to be kept. Combining this expression with Eq. (30), we get

$$\tilde{G}(p, 0) \simeq \frac{1}{M_*^4 \sqrt{\epsilon} 4\pi \left(1 + p \left(\frac{\epsilon}{M_*^2} \right)^{\frac{1}{2}} \right) + M_{\text{Pl}}^2 p^2}.$$

When the $M_{\text{Pl}}^2 p^2$ term in the denominator dominates this gives the 4D Green's function. The potential mediated by the scalar field Φ on the 4D worldvolume of the brane is determined as

$$V(r) = \int G(t, \vec{x}, y = 0; 0, 0, 0) dt,$$

where $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$. Using Eq. (32), we get

$$V(r) = \int_0^\infty \frac{dp}{(2\pi)^2} \frac{2p \sin(pr)}{r} \frac{1}{M_*^4 \sqrt{\epsilon} 4\pi \left(1 + p \left(\frac{\epsilon}{M_*^2} \right)^{\frac{1}{2}} \right) + M_{\text{Pl}}^2 p^2}. \quad (33)$$

⁸Note that for $N > 3$, in order to regularize the function D we should include higher order operators beyond the one included already in (24).

This potential behaves as that of a 4D theory when $\sqrt{\epsilon}/M_* \ll r \ll M_{\text{Pl}}/M_*^2$ and behaves as the potential of a $(4 + N)$ -dimensional theory for bigger values of r . The similar calculation for the crossover distance can be done for other N 's. The qualitative result is the same, i.e., the crossover distance is approximated by:

$$r_c^{(D \geq 6)} \sim \frac{M_{\text{Pl}}}{M_*^2}.$$

A simple estimate with $M_{\text{Pl}} \sim 10^{18}\text{GeV}$ and $M \sim 10^{-3}\text{eV}$ gives $r_c \sim 10^{29}\text{mm}$. At the distance smaller than r_c we will observe the four-dimensional world. Thus, this model predicts the *simultaneous* modification of gravity at the distance of order 0.1 millimeter and at around the Hubble distance!

Let us now study the case of two sources placed at different positions y and y' in the extra dimensions. The $N = 1$ theory was studied in [4] (see also [26]). In the present case the equation for the Green's function in the momentum four-space takes the form:

$$\left(M_*^5 \left[(p^2 - \partial_y^2) + \frac{\epsilon}{M_*^2} (p^2 - \partial_y^2)^2 \right] + M_{\text{Pl}}^2 \delta^{(3)}(y) p^2 \right) \tilde{G}(p, y, y') = \delta^{(3)}(y - y'). \quad (34)$$

The solution of Eq. (34) can be written as follows:

$$\begin{aligned} \tilde{G}(p, y, y') &= \frac{1}{M_*^5} \left(\frac{e^{-p|y-y'|} - e^{-|y-y'|\sqrt{p^2 + \frac{M_*^2}{\epsilon}}}}{4\pi |y - y'|} \right) \\ &- \frac{M_{\text{Pl}}^2 p^2}{M_*^5} \left(\frac{e^{-p|y|} - e^{-|y|\sqrt{p^2 + \frac{M_*^2}{\epsilon}}}}{4\pi |y|} \right) \times \left(\frac{e^{-p|y'|} - e^{-|y'|\sqrt{p^2 + \frac{M_*^2}{\epsilon}}}}{4\pi |y'|} \right) \\ &\times \frac{1}{M_*^5 + M_{\text{Pl}}^2 p^2 \left(\sqrt{p^2 + \frac{M_*^2}{\epsilon}} - p \right) / 4\pi}. \end{aligned} \quad (35)$$

Using this expression one finds the potential:

$$V(r, y, y') = \int G(t, \overrightarrow{x - x'}, y, y') dt = \int_0^\infty \frac{dp}{(2\pi)^2} \frac{2p \sin(pr)}{r} \tilde{G}(p, y, y'), \quad (36)$$

where $\tilde{G}(p, y, y')$ is given by Eq. (35). All the expressions above should be understood as an expansion in powers of (p^2/M_*^2) . The expression (35) reveals that the bulk interactions near the brane are strongly affected by the presence of the brane-induced terms. This is similar to what was observed in the $N = 1$ case in Ref. [4].

From Eqs. (36,35), it is clear that the potential between two sources placed at different positions in the extra dimensions does not give rise to any ultra-local interactions [26].

5 Propagators on Thick Branes

In this section we return to the consideration of branes with a finite width. We will check whether the results obtained for a delta function brane can be reproduced from a thick brane by the limiting procedure when $\Delta \rightarrow 0$.

The equation for the Green's function which is obtained from (24) takes the form:

$$\hat{\mathcal{D}} G(p, y) + M_{\text{Pl}}^2 p^2 f^2(y) \int d^N y' f^2(y') G(p, y') = g^2(y), \quad (37)$$

where f^2 and g^2 are two localized functions with different thicknesses (we drop the tilde sign over the momentum space Green's functions). $\hat{\mathcal{D}}$ stands for an operator in N dimensions which, as in the previous section, is approximated by the standard d'Alambertian plus one leading HD term:

$$\hat{\mathcal{D}} = M_*^{2+N} \left\{ p^2 - \nabla_N^2 + \frac{\epsilon}{M_*^2} (p^2 - \nabla_N^2)^2 \right\}. \quad (38)$$

Before we turn to the calculation of the Green's function let us make comments regarding the functions f and g . The former determines the localization of the breathing mode on the brane worldvolume while the latter is a measure of localization of a source under the consideration. In general these functions might be different. On the other hand, if we turn to the limit $M_* \rightarrow 0$, then Eq. (37) makes sense only if $f \rightarrow g$ in that limit. Thus f and g cannot be completely arbitrary functions, there should exist some correlation between them. This is a self-consistency requirement which will play a crucial role below.

We look for a solution of Eq. (37) in the following form:

$$G(p, y) = G_0(p, y) + \chi(p, y), \quad (39)$$

such that $G_0(p, y)$ obeys the equation

$$\hat{\mathcal{D}} G_0(p, y) = g^2(y), \quad (40)$$

and hence can be written as follows:

$$G_0(p, y) = \int d^N y' g^2(y') D_*(p, y - y'). \quad (41)$$

Here $D_* = D/M_*^{2+N}$, denotes the Green's function for the operator $\hat{\mathcal{D}}$ (D is defined in (29)). Note also that in the above expression for G_0 we neglected the zero-mode solution of the operator $\hat{\mathcal{D}}$ which in this case is unphysical.

The equation for the function $\chi(p, y)$ takes the form:

$$\hat{\mathcal{D}} \chi(p, y) + M_{\text{Pl}}^2 p^2 [Y_1 + Y_2] f^2(y) = 0, \quad (42)$$

where we introduced the following notations:

$$Y_1 \equiv \int d^N y' f^2(y') G_0(p, y'); \quad Y_2 \equiv \int d^N y' f^2(y') \chi(p, y'). \quad (43)$$

Disregarding the unphysical solution of the homogeneous equation we obtain:

$$\chi(p, y) = M_{\text{Pl}}^2 p^2 [Y_1 + Y_2] \int d^N y' f^2(y') D_*(p, y - y'). \quad (44)$$

In order to determine Y_2 we integrate this equation with the weight f^2 . Using the obtained expression we derive the equation for χ :

$$\chi(p, y) = - \frac{M_{\text{Pl}}^2 p^2 [\int d^N y' f^2(y') G_0(p, y')] [\int d^N z f^2(z) D_*(p, y - z)]}{1 + M_{\text{Pl}}^2 p^2 \int d^N y d^N y' f^2(y) D_*(p, y - y') f^2(y')}. \quad (45)$$

Therefore, the propagator in the general form reads as follows:

$$G(p, y) = G_0(p, y) - \frac{M_{\text{Pl}}^2 p^2 Y_1}{1 + M_{\text{Pl}}^2 p^2 Y_3} Y(p, y), \quad (46)$$

where we have used the notations of (43) and

$$\begin{aligned} Y_3 &\equiv \int d^N y d^N y' f^2(y) D_*(p, y - y') f^2(y'), \\ Y(p, y) &\equiv \int d^N y' f^2(y') D_*(p, y - y'). \end{aligned} \quad (47)$$

Below we will use these general expressions for the Green's function to derive the properties of the potential on the brane.

Let us study the $D = 5$ case first. We parametrize the functions f and g as follows:

$$f^2(y) = \begin{cases} \frac{1}{2\Delta} & \text{for } |y| < \Delta, \\ 0 & \text{for } |y| > \Delta; \end{cases} \quad g^2(y) = \begin{cases} \frac{1}{2\alpha} & \text{for } |y| < \alpha, \\ 0 & \text{for } |y| > \alpha. \end{cases} \quad (48)$$

First, for simplicity we assume that $\alpha < \Delta$. Then, inside the brane ($y < \Delta$) we find

$$\begin{aligned} Y_1 &= \frac{1}{2\Delta M_*^3 p^2} \left(1 - e^{-p\Delta} \frac{\sinh(p\alpha)}{p\alpha} \right), \\ Y_3 &= \frac{1}{2\Delta M_*^3 p^2} \left(1 - e^{-p\Delta} \frac{\sinh(p\Delta)}{p\Delta} \right), \\ Y(y) &= \frac{1}{2\Delta M_*^3 p^2} \left(1 - e^{-p\Delta} \cosh(py) \right). \end{aligned} \quad (49)$$

At $y = 0$ the first order correction to the propagator reads

$$G(p, 0) \simeq \frac{1}{M_{\text{Pl}}^2 p^2} \left(1 + \frac{p\Delta}{6} \right). \quad (50)$$

As we discussed above, the self-consistency of the equations for the Green's function (37) requires that $\Delta = \alpha + \mathcal{O}(M_*/M_{\text{Pl}}^2)$. This was used above and the leading term was kept. Thus, the correction due to the 4D propagator appears at the scale Δ .

However, there are other corrections due to the terms of order $\mathcal{O}(M_*/M_{\text{Pl}}^2)$ which distinguish α and Δ . These terms can give rise to the modification of the propagator at the momenta above M_* . The latter modification is expected to be in the theory anyway since the higher dimensional operators are suppressed by the scale M_* .

As an representative example of higher dimensions we again concentrate on the codimension three case, $N = 3$. For the functions f and g we choose the following interpolation:

$$f^2(y) = \begin{cases} \frac{3}{4\pi\Delta^3} & \text{for } |\vec{y}| < \Delta, \\ 0 & \text{for } |\vec{y}| > \Delta; \end{cases} \quad g^2(y) = \begin{cases} \frac{3}{4\pi\alpha^3} & \text{for } |\vec{y}| < \alpha, \\ 0 & \text{for } |\vec{y}| > \alpha. \end{cases} \quad (51)$$

As before we set $\alpha < \Delta$. Somewhat lengthy calculation gives the following result for the Green's function expanded in powers of the brane width Δ :

$$G(p, 0) \simeq \frac{1}{M_{\text{Pl}}^2 p^2} \left(1 + \frac{39}{140} p \Delta \right). \quad (52)$$

As in the previous case we used the consistency relation $\Delta = \alpha + \mathcal{O}(M_*/M_{\text{Pl}}^2)$. The corrections to the 4D behavior arise at the scale of order $1/\Delta$. However, this might be changed if we were to take into account a possible difference between Δ and α which is of order $\mathcal{O}(M_*/M_{\text{Pl}}^2)$. The latter results in the corrections to the propagator which become significant at the scale M_* . Hence, the propagators in $D = 5$ and $D \geq 6$ is rather similar.

6 The Crossover Scale

In this section we will summarize the results on the crossover scale r_c . This quantity is determined by the 4D Planck mass M_{Pl} and the bulk Planck mass M_* . As we discussed before, in a 5D theory with a brane-induced Ricci term the expression takes the form:

$$r_c^{(D=5)} \sim \frac{M_{\text{Pl}}^2}{M_*^3}. \quad (53)$$

While in the case of higher dimensions with zero-tension branes and the HD operators suppressed by powers of $1/M_*$ we get:

$$r_c^{(D \geq 6)} \sim \frac{M_{\text{Pl}}}{M_*^2}. \quad (54)$$

Thus, in higher dimensions $D \geq 6$ the value of r_c is smaller.

Let us now discuss what are the phenomenological bounds on r_c ? The bounds could come from different sources. Let us start with the upper bound. There is a lower bound on the value of M_* which comes from the data on sub-millimeter gravity measurements [1] and the accelerator, astrophysical and cosmological data, that is $M_* \gtrsim 10^{-3}$ eV [4]. Hence, the upper bounds on r_c are:

$$r_c^{(D=5)} \lesssim 10^{59} \text{ mm}; \quad r_c^{(D \geq 6)} \lesssim 10^{29} \text{ mm}. \quad (55)$$

As we discussed in the introduction, the latter number coincides with the present day horizon size. Therefore, the model in $D \geq 6$ with $M_* \sim 10^{-3}$ eV predicts simultaneous modification of laws of gravity at short distances around 0.1 mm and at large distances around the Hubble scale.

Let us now turn to the lower bound on the crossover scale. This bound can come from the measurements of the Newton force at macroscopic distances as well as from the cosmological considerations.

The distance at which Newton's law is known to hold exceeds somewhat the solar system size. Beyond this distance scale, because of the presence of dark matter, the Newton law can be modified if the simultaneous changes are made in the amount and distribution of dark matter so that these changes render a theory consistent with the data on large scale structure. Therefore, from these arguments:

$$r_c^{\text{exp}} \gg 10^{15} \text{ cm} . \quad (56)$$

However, the cosmological consideration can impose the stringer bound. Indeed, if the cosmological evolution changes at distances of order r_c , then we want r_c to be of the order of the Hubble scale at least. The cosmological solution of the 5D theory which changes the regime as the Hubble parameter becomes of order r_c was found by Deffayet in Ref. [6]. Therefore, we have to impose $r_c^{\text{exp}} \gtrsim 10^{29}$ mm. In this case, in the 5D theory we get 10^{-3} eV $\lesssim M_* \lesssim 10^7$ eV. However, in the $D \geq 6$ case we obtain a very stringent restriction, $M_* \sim 10^{-3}$ eV. Thus, the fundamental scale of gravity in this model is bound to be $M_* \sim 10^{-3}$ eV.

Before we turn to the next section we would like to make comments on some new developments. Recently new interesting branches of cosmological solutions were found by Dick [8, 9], and by Cordero and Vilenkin [10]. The creation of the “stealth” branes were discussed in [10]. These solutions, from the perspective of a 4D braneworld observer, coincide with the well known solutions of pure 4D theory. On this branch, there is no difference between the time evolution of the universe in just a purely 4D theory from that in the higher dimensional theory discussed in the present paper. These solutions are found as follows [8–10]. Consider the Einstein equations with the induced terms included:

$$\begin{aligned} M_*^{2+N} \left(\mathcal{R}_{AB} - \frac{1}{2} G_{AB} \mathcal{R} \right) + M_{\text{ind}}^2 \delta_A^\mu \delta_B^\nu \delta^{(N)}(y) \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) \\ = \delta_A^\mu \delta_B^\nu T_{\mu\nu}^{\text{Brane}} \delta^{(N)}(y) . \end{aligned} \quad (57)$$

Then, there can exist a solution which is Riemann-flat in the bulk but satisfies the equation on the brane, i.e.:

$$\begin{aligned} \mathcal{R}_{AB} &= 0 , \\ M_{\text{ind}}^2 \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) &= T_{\mu\nu}^{\text{Brane}} . \end{aligned} \quad (58)$$

The solution of the second equation in this system is exactly the one for a pure 4D theory. It is also a solution to the whole 5D system provided that the metric is Riemann-flat in higher dimensions see Refs. [8, 9], [10].

If this branch is realized, then there is no bound on r_c arising from cosmology. The only restriction on r_c comes [9] from the solar system measurements of Newton's law (56). This can be translated into the following upper bounds on M_* :

$$\begin{aligned} 10^{-3} \text{ eV} &\lesssim M_* \lesssim 10^{12} \text{ eV} , & \text{for } D = 5 ; \\ 10^{-3} \text{ eV} &\lesssim M_* \lesssim 10^4 \text{ eV} , & \text{for } D \geq 6 . \end{aligned} \quad (59)$$

As before, there is a bigger range for M_* in the 5D theory.

7 Nonzero Tension Branes

In the previous sections we dealt with branes (or system of branes) which were placed at orbifold fixed points and for which the tension effects were suppressed by fine tuning. Here we would like to study the influence of a nonzero brane tension on the expression for r_c . We start with an infinite $(4 + N)$ dimensional space. Consider a certain brane in this space which has an intrinsic tension T . The brane distorts the ambient space around it. For a BPS D3-brane the solution is regular outside of the core and has well defined horizon. However, for non-BPS branes which are phenomenologically relevant, the solutions give rise to singularities. These singularities are hoped to be artifacts of the delta-function approximation and are expected to be removed if the core of the brane is smoothed [30] (see also [31]). We will adopt below this philosophy.

For a brane world which preserves 4D Lorentz invariance the interval takes the form:

$$ds^2 = A^2(y) \eta_{\mu\nu} dx^\mu dx^\nu - B^2(y) dy^2 - C^2(y) y^2 d\Omega_{N-1}^2 , \quad (60)$$

where the functions A , B and C depend on the brane tension and M_* . For certain cases A , B and C are known exactly (see, e.g., Refs. [32, 30, 33]). In the $N = 2$ case a positive-tension brane creates a deficit angle and produces no other nontrivial warp-factors like A , B and C . The effects of this on the model with brane-induced Einstein-Hilbert term were studied in Ref. [34] where it was concluded that the qualitative behavior found in [5] (and outlined above) does not change.

Below we will concentrate on the case $N \geq 3$. We use the solutions for non-BPS branes obtained in Ref. [30]:

$$A = f^{-\frac{1}{4}} \sqrt{\frac{N-1}{N+2}} , \quad B = f^{-\frac{1}{N-2}} \left(\frac{N-3}{2} - \sqrt{\frac{N-1}{N+2}} \right) , \quad C = f^{\frac{1}{N-2}} \left(\frac{1}{2} + \sqrt{\frac{N-1}{N+2}} \right) , \quad (61)$$

where

$$f \equiv 1 + \left(\frac{y_g}{y} \right)^{N-2} . \quad (62)$$

Here the gravitational radius of a brane in the transverse direction y_g is determined by the brane tension T and can be written as follows

$$y_g = \left(\frac{T}{M_*^4} \right)^{\frac{1}{N-2}} \frac{1}{M_*} . \quad (63)$$

In order to determine the properties of the interactions on the brane one needs to calculate a graviton propagator on the background (61). The exact calculation is cumbersome (for certain calculations along these directions see Ref. [31]). However, one can do a qualitative analysis of the equation for perturbations. The analysis is similar to that performed in Ref. [5] and will not be repeated here. We just mention the basic steps. The solutions for A, B and C in (61) are singular near $y = 0$. However, in this domain the classical theory breaks down and the expressions (61) should be smoothed out. In the previous section we accounted for this UV regularization by introducing higher dimensional operators. This is perhaps the most consistent and reliable way of dealing with the singularities. However, when a nonzero brane tension is taken into account, the higher dimensional operators complicate the equations even further so that they are not treatable analytically. Under these circumstances, it becomes more reasonable to introduce other UV regularization, e.g., one could use the finite width of the brane core Δ . Then, we can estimate the critical momentum/distance for which the 4D induced term becomes dominant over the bulk terms in the background (61). This consideration leads to the following region where the effects of the 4D induced terms dominate:

$$r \lesssim r_c^{T \neq 0} \sim \frac{M_{\text{Pl}}}{\sqrt{T + \Delta^{N-2} M_*^{N+2}}} . \quad (64)$$

Here the 4D Planck mass is determined by the product of the induced scale and the value of the function A taken in the core $M_{\text{Pl}}^2 = M_{\text{ind}}^2 A^2(\Delta)$. Although, the value of $A^2(\Delta)$ is not known, one might expect that it is not far away from the value of this function at the point where the gravity approximation is reliable. Thus, it adds/subtracts an order of magnitude from M_{ind} .

Let us now consider various limits in (64).

First of all we notice that in order for the model to be realistic, i.e., the $r_c^{T \neq 0}$ be consistent with (56), there is an absolute upper bound on the value of the tension of the brane, $T \leq (10^4 \text{ eV})^4$. This is a rather low tension from the particle physics point of view. Therefore, certain fine tuning of the 4D cosmological constant is needed in order to satisfy this relation. The cosmological considerations discussed in the previous section, however, impose even more restrictive bound on the tension $T \leq (10^{-3} \text{ eV})^4$. This, needs fine tuning of the 4D cosmological constant.

Let us now turn to the constructions in which our brane is represented by a stack of branes and anti-branes (or orientifold planes) which can be placed at some orbifold fixed points. In this case one can study consistently the limit $T \rightarrow 0$. The resulting expression in (64) coincides with that obtained in Ref. [5]. The crossover scale depends in general on the number of dimensions. On the other hand, in the previous sections we studied a theory with high derivatives and obtained the crossover scale which was independent of N (54). The question is how these two results are reconciled. The reconciliation is based on the following arguments. When we take into account the higher dimensional operators which are suppressed by M_* , this means that UV gravitational resolution of the brane width is at the scale

$\Delta \sim 1/M_*$. Substituting the latter expression in Eq. (64) with $T = 0$ we recover Eq. (54). Thus, the phenomenological constraints on the bulk gravitational scale are the same as in the previous section.

Finally, the following scenario is also possible: suppose the bulk gravitational scale is high, let us say in a TeV region or so. Let us call this scale $M_{\text{Bulk}} \gtrsim \text{TeV}$. It is conceivable that some scalar field dynamics suppresses the coefficient in front of the Ricci scalar in the bulk. Then, $M_*^{4+N} = f(\langle\Phi\rangle)M_{\text{Bulk}}^{4+N}$, where $f(\langle\Phi\rangle)$ is a suppression factor which depends on the vacuum expectation value of a scalar. On the other hand, the higher derivative terms in the bulk are suppressed by the scale $M_{\text{Bulk}} \gg M_*$. Let us consider a zero-tension brane at the orbifold fixed point. Then, in Eq. (64) we should take $\Delta \sim 1/M_{\text{Bulk}}$. Using the hierarchy $M_{\text{Bulk}} \gg M_*$ we can obtain an acceptable values of r_c even for $M_* \sim \text{TeV}$. For instance if $M_{\text{Bulk}} \sim M_{\text{Pl}}$ we get the value of r_c which is consistent with (56) for $N \geq 4$.

8 Conclusions

Summarizing, we discussed a brane-world model with two or more infinite-volume extra dimensions. This model predicts the simultaneous modification of the law of gravity at around 0.1 mm and at the Hubble scale. Therefore, the fundamental scale of gravity is bound to be 10^{-3} eV in this model. These predictions can be tested by table-top gravitational measurements and by astrophysical experiments.

The UV singularities which arise in this model can be treated consistently. We used both, the regularization by a finite brane width, and by the higher-dimensional operators which are suppressed by the fundamental scale of gravity. We calculated the corresponding two-point Green's functions in the regularized theory and showed that there is an intimate relation between the UV and IR scales where gravity changes its regime.

9 Acknowledgments

We are grateful to S. Dimopoulos, A. Gruzinov, D. Kabat, F. Nitti, M. Perelstein, R. Rattazzi, M. Shifman, A. Vainshtein, M. Voloshin and M. Wise for useful discussions. GD and GG thank *The Aspen Center for Physics*, and GG thanks CERN Theory Division for the hospitality where parts of this work were done. The work of GD was supported in part by David and Lucille Packard Foundation Fellowship for Science and Engineering, by Alfred P. Sloan foundation fellowship and by NSF grant PHY-0070787. The work of GG and XH is supported by DOE Grant DE-FG02-94ER408.

References

- [1] C. D. Hoyle, U. Schmidt, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, D. J. Kapner and H. E. Swanson, Phys. Rev. Lett. **86**, 1418 (2001) [arXiv:hep-ph/0011014];
J.C. Price, in *Proceedings of International Symposium on Experimental Gravitational Physics*, ed. Michelson, Guangzhou, China (World Scientific, Singapore, 1988);
J. Long, “Laboratory Search for Extra-Dimensional Effects in Sub-Millimeter Regime” Talk given at the International Conference on Physics Beyond Four Dimensions, ICTP, Trieste, Italy; July 3-6, (2000);
A. Kapitulnik, “Experimental Tests of Gravity Below 1mm” Talk given at the International Conference on Physics Beyond Four Dimensions, ICTP, Trieste, Italy; July 3-6, (2000) .
- [2] A.G. Riess et al., *Astroph. J* **116**, 1009 (1998);
S. Perlmutter et al., “Measurements of Omega and Lambda from 42 High-Redshift Supernovae”, [astro-ph/9812133];
A.G. Riess, Talk Given at The Symposium “*The Dark Universe: Matter, Energy, and Gravity*” Baltimore, April 2-5, (2001).
- [3] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000) [arXiv:hep-th/0005016].
- [4] G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, arXiv:hep-th/0106058; Phys. Rev. D **64**, 084004 (2001) [arXiv:hep-ph/0102216].
- [5] G. R. Dvali and G. Gabadadze, Phys. Rev. D **63**, 065007 (2001) [arXiv:hep-th/0008054].
- [6] C. Deffayet, Phys. Lett. B **502**, 199 (2001) [arXiv:hep-th/0010186].
- [7] C. Deffayet, G. R. Dvali and G. Gabadadze, arXiv:astro-ph/0105068; arXiv:astro-ph/0106449.
- [8] R. Dick, Class. Quant. Grav. **18**, R1 (2001) [arXiv:hep-th/0105320].
- [9] R. Dick, arXiv:hep-th/0110162.
- [10] R. Cordero and A. Vilenkin, arXiv:hep-th/0107175.
- [11] G. Kofinas, JHEP **0108**, 034 (2001) [arXiv:hep-th/0108013].
- [12] C. Deffayet, G. R. Dvali, G. Gabadadze and A. I. Vainshtein, arXiv:hep-th/0106001.
- [13] I. Giannakis and H. c. Ren, arXiv:hep-th/0111127.

- [14] A. Lue, arXiv:hep-th/0111168.
- [15] G. R. Dvali, G. Gabadadze and M. A. Shifman, Phys. Lett. B **497**, 271 (2001) [arXiv:hep-th/0010071].
- [16] E. K. Akhmedov, Phys. Lett. B **521**, 79 (2001) [arXiv:hep-th/0107223].
- [17] I. I. Kogan, S. Mouslopoulos, A. Papazoglou, G. G. Ross and J. Santiago, Nucl. Phys. B **584**, 313 (2000) [arXiv:hep-ph/9912552].
- [18] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, Phys. Rev. Lett. **84**, 5928 (2000) [arXiv:hep-th/0002072].
- [19] R. Sundrum, JHEP **9907**, 001 (1999) [hep-ph/9708329].
- [20] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **484**, 112 (2000) [arXiv:hep-th/0002190]; Phys. Lett. B **484**, 129 (2000) [arXiv:hep-th/0003054].
- [21] E. Witten, “The cosmological constant from the viewpoint of string theory,” *Lecture given at 4th International Symposium on Sources and Detection of Dark Matter in the Universe*; Marina del Rey, California, 23-25 Feb 2000. [arXiv:hep-ph/0002297].
- [22] E. Kiritsis, N. Tetradis and T. N. Tomaras, JHEP **0108**, 012 (2001) [arXiv:hep-th/0106050].
- [23] S. Corley, D. A. Lowe and S. Ramgoolam, JHEP **0107**, 030 (2001) [arXiv:hep-th/0106067].
- [24] Z. Kakushadze, JHEP **0110**, 031 (2001) [arXiv:hep-th/0109054].
- [25] M. Carena, A. Delgado, J. Lykken, S. Pokorski, M. Quiros and C. E. Wagner, Nucl. Phys. B **609**, 499 (2001) [arXiv:hep-ph/0102172].
- [26] S. L. Dubovsky and V. A. Rubakov, Int. J. Mod. Phys. A **16**, 4331 (2001) [arXiv:hep-th/0105243].
- [27] D.M. Capper, Nuovo Cim. **A25** 29 (1975).
- [28] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994) [arXiv:gr-qc/9405057].
J. F. Donoghue, “Introduction to the Effective Field Theory Description of Gravity,” *Talk given at Advanced School on Effective Theories*, Almunecar, Spain, 25 Jun - 1 Jul 1995. [arXiv:gr-qc/9512024].
- [29] A.I. Vainshtein, Phys. Lett. **39B**, 393 (1972) .
- [30] R. Gregory, Nucl. Phys. B **467**, 159 (1996) [arXiv:hep-th/9510202].

- [31] C. Charmousis, R. Emparan and R. Gregory, JHEP **0105**, 026 (2001) [arXiv:hep-th/0101198].
- [32] M. J. Duff, R. R. Khuri and J. X. Lu, Phys. Rept. **259**, 213 (1995) [arXiv:hep-th/9412184].
- [33] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111].
- [34] O. Corradini, A. Iglesias, Z. Kakushadze and P. Langfelder, Phys. Lett. B **521**, 96 (2001) [arXiv:hep-th/0108055].